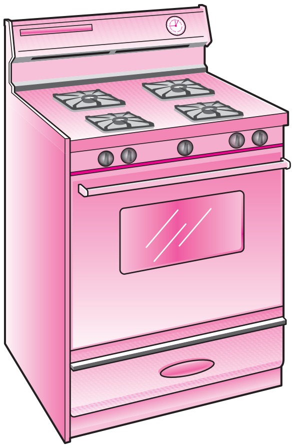

FIGURE 2–9

Heat is generated in the heating coils of an electric range as a result of the conversion of electrical energy to heat.

Heat Generation

A medium through which heat is conducted may involve the conversion of mechanical, electrical, nuclear, or chemical energy into heat (or thermal energy). In heat conduction analysis, such conversion processes are characterized as **heat** (or **thermal energy**) **generation**.

For example, the temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at a rate of I^2R, where I is the current and R is the electrical resistance of the wire (Fig. 2–9). The safe and effective removal of this heat away from the sites of heat generation (the electronic circuits) is the subject of *electronics cooling,* which is one of the modern application areas of heat transfer.

Likewise, a large amount of heat is generated in the fuel elements of nuclear reactors as a result of nuclear fission that serves as the *heat source* for the nuclear power plants. The natural disintegration of radioactive elements in nuclear waste or other radioactive material also results in the generation of heat throughout the body. The heat generated in the sun as a result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth.

Another source of heat generation in a medium is exothermic chemical reactions that may occur throughout the medium. The chemical reaction in this case serves as a *heat source* for the medium. In the case of endothermic reactions, however, heat is absorbed instead of being released during reaction, and thus the chemical reaction serves as a *heat sink.* The heat generation term becomes a negative quantity in this case.

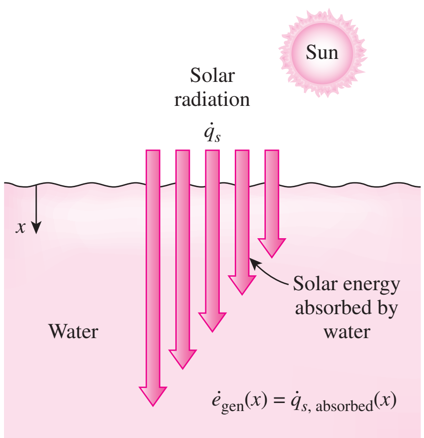

FIGURE 2–10

The absorption of solar radiation by water can be treated as heat generation.

Often it is also convenient to model the absorption of radiation such as solar energy or gamma rays as heat generation when these rays penetrate deep into the body while being absorbed gradually. For example, the absorption of solar energy in large bodies of water can be treated as heat generation throughout the water at a rate equal to the rate of absorption, which varies with depth (Fig. 2–10). But the absorption of solar energy by an opaque body occurs within a few microns of the surface, and the solar energy that penetrates into the medium in this case can be treated as specified heat flux on the surface.

Note that heat generation is a *volumetric phenomenon.* That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified *per unit volume* and is denoted by $\dot{e}_{gen}$, whose unit is W/m^3 or $Btu/h{\cdot}ft^3$.

The rate of heat generation in a medium may vary with time as well as position within the medium. When the variation of heat generation with position is known, the *total* rate of heat generation in a medium of volume V can be determined from

$$\dot{E}_{gen} = \int_V \dot{e}_{gen}\, dV \qquad \text{(W)} \tag{2–5}$$

In the special case of *uniform* heat generation, as in the case of electric resistance heating throughout a homogeneous material, the relation in Eq. 2–5 reduces to $\dot{E}_{gen} = \dot{e}_{gen}V$, where $\dot{e}_{gen}$ is the constant rate of heat generation per unit volume.

EXAMPLE 2–1 Heat Generation in a Hair Dryer

The resistance wire of a 1200-W hair dryer is 80 cm long and has a diameter of $D = 0.3$ cm (Fig. 2–11). Determine the rate of heat generation in the wire per unit volume, in W/cm^3, and the heat flux on the outer surface of the wire as a result of this heat generation.

SOLUTION The power consumed by the resistance wire of a hair dryer is given. The heat generation and the heat flux are to be determined.

Assumptions Heat is generated uniformly in the resistance wire.

Analysis A 1200-W hair dryer converts electrical energy into heat in the wire at a rate of 1200 W. Therefore, the rate of heat generation in a resistance wire is equal to the power consumption of a resistance heater. Then the rate of heat generation in the wire per unit volume is determined by dividing the total rate of heat generation by the volume of the wire,

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{(\pi D^2/4)L} = \frac{1200 \text{ W}}{[\pi(0.3 \text{ cm})^2/4](80 \text{ cm})} = \mathbf{212 \text{ W/cm}^3}$$

Similarly, heat flux on the outer surface of the wire as a result of this heat generation is determined by dividing the total rate of heat generation by the surface area of the wire,

$$\dot{Q}_s = \frac{\dot{E}_{\text{gen}}}{A_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi DL} = \frac{1200 \text{ W}}{\pi(0.3 \text{ cm})(80 \text{ cm})} = \mathbf{15.9 \text{ W/cm}^3}$$

Discussion Note that heat generation is expressed per unit volume in W/cm^3 or Btu/h·ft^3, whereas heat flux is expressed per unit surface area in W/cm^2 or Btu/h·ft^2.

FIGURE 2–11
Schematic for Example 2–1.

2–2 ▪ ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

Consider heat conduction through a large plane wall such as the wall of a house, the glass of a single pane window, the metal plate at the bottom of a pressing iron, a cast-iron steam pipe, a cylindrical nuclear fuel element, an electrical resistance wire, the wall of a spherical container, or a spherical metal ball that is being quenched or tempered. Heat conduction in these and many other geometries can be approximated as being *one-dimensional* since heat conduction through these geometries is dominant in one direction and negligible in other directions. Next we develop the one-dimensional heat conduction equation in rectangular, cylindrical, and spherical coordinates.

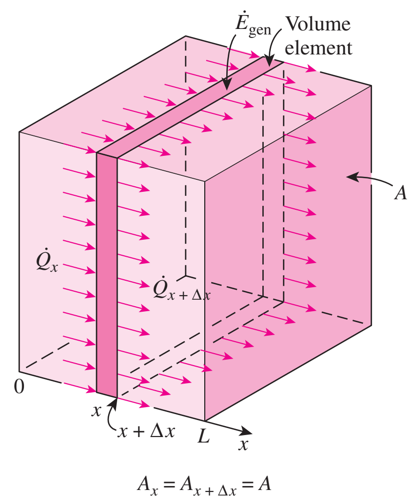

FIGURE 2–12
One-dimensional heat conduction through a volume element in a large plane wall.

Heat Conduction Equation in a Large Plane Wall

Consider a thin element of thickness Δx in a large plane wall, as shown in Fig. 2–12. Assume the density of the wall is ρ, the specific heat is c, and the area of the wall normal to the direction of heat transfer is A. An *energy balance* on this thin element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \tag{2–6}$$

But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta x (T_{t+\Delta t} - T_t) \tag{2–7}$$

$$\dot{E}_{\text{gen, element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta x \tag{2–8}$$

Substituting into Eq. 2–6, we get

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{e}_{\text{gen}} A \Delta x = \rho c A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2–9}$$

Dividing by $A\Delta$x gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2–10}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t} \tag{2–11}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{\partial \dot{Q}}{\partial x} = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right) \tag{2–12}$$

Noting that the area A is constant for a plane wall, the one-dimensional transient heat conduction equation in a plane wall becomes

Variable conductivity:
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t} \tag{2–13}$$

The thermal conductivity k of a material, in general, depends on the temperature T (and therefore x), and thus it cannot be taken out of the derivative. However, the *thermal conductivity* in most practical applications can be assumed to remain *constant* at some average value. The equation above in that case reduces to

Constant conductivity:
$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2–14}$$

where the property $\alpha = k/\rho c$ is the **thermal diffusivity** of the material and represents how fast heat propagates through a material. It reduces to the following forms under specified conditions (Fig. 2–13):

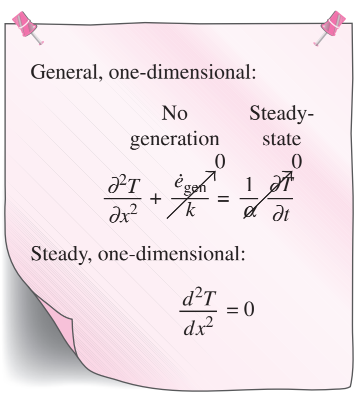

FIGURE 2–13

The simplification of the one-dimensional heat conduction equation in a plane wall for the case of constant conductivity for steady conduction with no heat generation.

(1) *Steady-state:* $(\partial/\partial t = 0)$

$$\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{k} = 0 \quad \text{(2–15)}$$

(2) *Transient, no heat generation:* $(\dot{e}_{gen} = 0)$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{(2–16)}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0 \text{ and } \dot{e}_{gen} = 0)$

$$\frac{d^2T}{dx^2} = 0 \quad \text{(2–17)}$$

Note that we replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case since the partial and ordinary derivatives of a function are identical when the function depends on a single variable only [$T = T(x)$ in this case].

Heat Conduction Equation in a Long Cylinder

Now consider a thin cylindrical shell element of thickness Δr in a long cylinder, as shown in Fig. 2–14. Assume the density of the cylinder is ρ, the specific heat is c, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r+\Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{gen,\ element} = \frac{\Delta E_{element}}{\Delta t} \quad \text{(2–18)}$$

The change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{element} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A\Delta r(T_{t+\Delta t} - T_t) \quad \text{(2–19)}$$

$$\dot{E}_{gen,\ element} = \dot{e}_{gen}V_{element} = \dot{e}_{gen}A\Delta r \quad \text{(2–20)}$$

Substituting into Eq. 2–18, we get

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{gen}A\Delta r = \rho c A\Delta r\frac{T_{t+\Delta t} - T_t}{\Delta t} \quad \text{(2–21)}$$

where $A = 2\pi rL$. You may be tempted to express the area at the *middle* of the element using the *average* radius as $A = 2\pi(r + \Delta r/2)L$. But there is nothing we can gain from this complication since later in the analysis we will take the limit as $\Delta r \to 0$ and thus the term $\Delta r/2$ will drop out. Now dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{gen} = \rho c\frac{T_{t+\Delta t} - T_t}{\Delta t} \quad \text{(2–22)}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{e}_{gen} = \rho c\frac{\partial T}{\partial t} \quad \text{(2–23)}$$

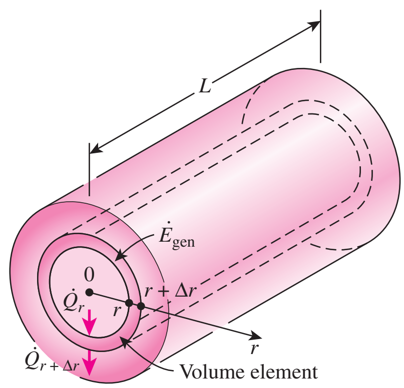

FIGURE 2–14

One-dimensional heat conduction through a volume element in a long cylinder.

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right) \tag{2–24}$$

Noting that the heat transfer area in this case is $A = 2\pi rL$, the one-dimensional transient heat conduction equation in a cylinder becomes

Variable conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{e}_{gen} = \rho c\frac{\partial T}{\partial t} \tag{2–25}$$

For the case of constant thermal conductivity, the previous equation reduces to

Constant conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2–26}$$

where again the property $\alpha = k/\rho c$ is the thermal diffusivity of the material. Eq. 2–26 reduces to the following forms under specified conditions (Fig. 2–15):

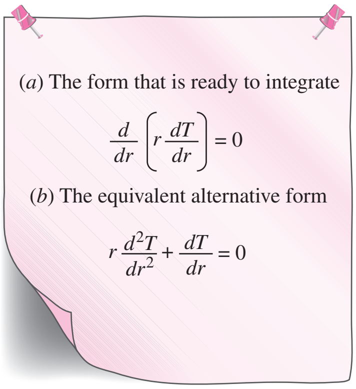

FIGURE 2–15
Two equivalent forms of the differential equation for the one-dimensional steady heat conduction in a cylinder with no heat generation.

(1) *Steady-state:* ($\partial/\partial t = 0$)
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{gen}}{k} = 0 \tag{2–27}$$

(2) *Transient, no heat generation:* ($\dot{e}_{gen} = 0$)
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2–28}$$

(3) *Steady-state, no heat generation:* ($\partial/\partial t = 0$ and $\dot{e}_{gen} = 0$)
$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \tag{2–29}$$

Note that we again replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case since the partial and ordinary derivatives of a function are identical when the function depends on a single variable only [$T = T(r)$ in this case].

Heat Conduction Equation in a Sphere

Now consider a sphere with density ρ, specific heat c, and outer radius R. The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$, where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case also, and thus it varies with location. By considering a thin spherical shell element of thickness Δr and repeating the approach described above for the cylinder by using $A = 4\pi r^2$ instead of $A = 2\pi rL$, the one-dimensional transient heat conduction equation for a sphere is determined to be (Fig. 2–16)

Variable conductivity:
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial T}{\partial r}\right) + \dot{e}_{gen} = \rho c\frac{\partial T}{\partial t} \tag{2–30}$$

which, in the case of constant thermal conductivity, reduces to

Constant conductivity:
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2–31}$$

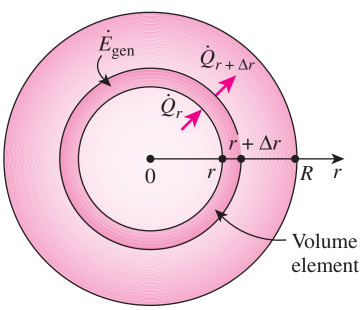

FIGURE 2–16
One-dimensional heat conduction through a volume element in a sphere.

where again the property $\alpha = k/\rho c$ is the thermal diffusivity of the material. It reduces to the following forms under specified conditions:

(1) *Steady-state:* $(\partial/\partial t = 0)$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0 \tag{2–32}$$

(2) *Transient, no heat generation:* $(\dot{e}_{\text{gen}} = 0)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2–33}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{e}_{\text{gen}} = 0)$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \quad \text{or} \quad r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0 \tag{2–34}$$

where again we replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case.

Combined One-Dimensional Heat Conduction Equation

An examination of the one-dimensional transient heat conduction equations for the plane wall, cylinder, and sphere reveals that all three equations can be expressed in a compact form as

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n k\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c\frac{\partial T}{\partial t} \tag{2–35}$$

where $n = 0$ for a plane wall, $n = 1$ for a cylinder, and $n = 2$ for a sphere. In the case of a plane wall, it is customary to replace the variable r by x. This equation can be simplified for steady-state or no heat generation cases as described before.

EXAMPLE 2–2 **Heat Conduction through the Bottom of a Pan**

Consider a steel pan placed on top of an electric range to cook spaghetti (Fig. 2–17). The bottom section of the pan is 0.4 cm thick and has a diameter of 18 cm. The electric heating unit on the range top consumes 800 W of power during cooking, and 80 percent of the heat generated in the heating element is transferred uniformly to the pan. Assuming constant thermal conductivity, obtain the differential equation that describes the variation of the temperature in the bottom section of the pan during steady operation.

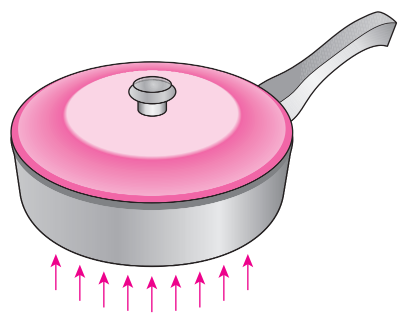

FIGURE 2–17
Schematic for Example 2–2.

SOLUTION A steel pan placed on top of an electric range is considered. The differential equation for the variation of temperature in the bottom of the pan is to be obtained.

Analysis The bottom section of the pan has a large surface area relative to its thickness and can be approximated as a large plane wall. Heat flux is applied to the bottom surface of the pan uniformly, and the conditions on the inner surface are also uniform. Therefore, we expect the heat transfer through the bottom section of the pan to be from the bottom surface toward the top, and heat transfer in this case can reasonably be approximated as being one-dimensional. Taking the direction normal to the bottom surface of the pan to be the x-axis, we will have $T = T(x)$ during steady operation since the temperature in this case will depend on x only.

The thermal conductivity is given to be constant, and there is no heat generation in the medium (within the bottom section of the pan). Therefore, the differential equation governing the variation of temperature in the bottom section of the pan in this case is simply Eq. 2–17,

$$\frac{d^2T}{dx^2} = 0$$

which is the steady one-dimensional heat conduction equation in rectangular coordinates under the conditions of constant thermal conductivity and no heat generation.

Discussion Note that the conditions at the surface of the medium have no effect on the differential equation.

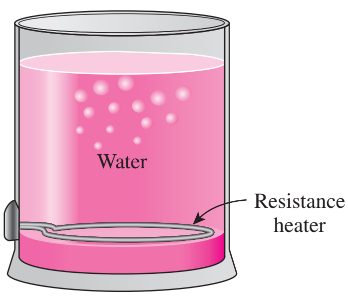

FIGURE 2–18

Schematic for Example 2–3.

EXAMPLE 2–3 Heat Conduction in a Resistance Heater

A 2-kW resistance heater wire with thermal conductivity $k = 15$ W/m·K, diameter $D = 0.4$ cm, and length $L = 50$ cm is used to boil water by immersing it in water (Fig. 2–18). Assuming the variation of the thermal conductivity of the wire with temperature to be negligible, obtain the differential equation that describes the variation of the temperature in the wire during steady operation.

SOLUTION The resistance wire of a water heater is considered. The differential equation for the variation of temperature in the wire is to be obtained.

Analysis The resistance wire can be considered to be a very long cylinder since its length is more than 100 times its diameter. Also, heat is generated uniformly in the wire and the conditions on the outer surface of the wire are uniform. Therefore, it is reasonable to expect the temperature in the wire to vary in the radial r direction only and thus the heat transfer to be one-dimensional. Then we have $T = T(r)$ during steady operation since the temperature in this case depends on r only.

The rate of heat generation in the wire per unit volume can be determined from

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{(\pi D^2/4)L} = \frac{2000 \text{ W}}{[\pi(0.004 \text{ m})^2/4](0.5 \text{ m})} = 0.318 \times 10^9 \text{ W/m}^3$$

Noting that the thermal conductivity is given to be constant, the differential equation that governs the variation of temperature in the wire is simply Eq. 2–27,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

which is the steady one-dimensional heat conduction equation in cylindrical coordinates for the case of constant thermal conductivity.

Discussion Note again that the conditions at the surface of the wire have no effect on the differential equation.

EXAMPLE 2–4 Cooling of a Hot Metal Ball in Air

A spherical metal ball of radius R is heated in an oven to a temperature of 600°F throughout and is then taken out of the oven and allowed to cool in ambient air at $T_\infty = 75°F$ by convection and radiation (Fig. 2–19). The thermal conductivity of the ball material is known to vary linearly with temperature. Assuming the ball is cooled uniformly from the entire outer surface, obtain the differential equation that describes the variation of the temperature in the ball during cooling.

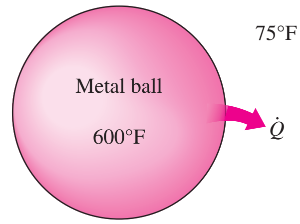

FIGURE 2–19
Schematic for Example 2–4.

SOLUTION A hot metal ball is allowed to cool in ambient air. The differential equation for the variation of temperature within the ball is to be obtained.

Analysis The ball is initially at a uniform temperature and is cooled uniformly from the entire outer surface. Also, the temperature at any point in the ball changes with time during cooling. Therefore, this is a one-dimensional transient heat conduction problem since the temperature within the ball changes with the radial distance r and the time t. That is, $T = T(r, t)$.

The thermal conductivity is given to be variable, and there is no heat generation in the ball. Therefore, the differential equation that governs the variation of temperature in the ball in this case is obtained from Eq. 2–30 by setting the heat generation term equal to zero. We obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial T}{\partial r}\right) = \rho c \frac{\partial T}{\partial t}$$

which is the one-dimensional transient heat conduction equation in spherical coordinates under the conditions of variable thermal conductivity and no heat generation.

Discussion Note again that the conditions at the outer surface of the ball have no effect on the differential equation.

2–3 ▪ GENERAL HEAT CONDUCTION EQUATION

In the last section we considered one-dimensional heat conduction and assumed heat conduction in other directions to be negligible. Most heat transfer problems encountered in practice can be approximated as being one-dimensional, and we mostly deal with such problems in this text. However, this is not always the case, and sometimes we need to consider heat transfer in other directions as well. In such cases heat conduction is said to be *multidimensional,* and in this section we develop the governing differential equation in such systems in rectangular, cylindrical, and spherical coordinate systems.

Rectangular Coordinates

Consider a small rectangular element of length Δx, width Δy, and height Δz, as shown in Fig. 2–20. Assume the density of the body is ρ and the specific heat is c. An *energy balance* on this element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ x,\ y,\text{ and } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y,\text{ and } z+\Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{pmatrix}$$

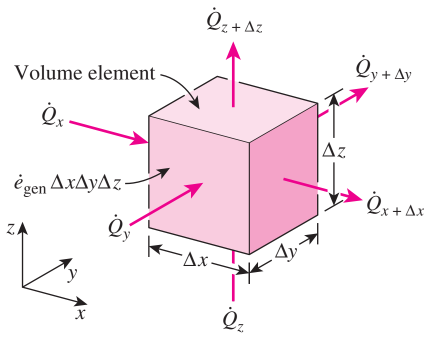

FIGURE 2–20
Three-dimensional heat conduction through a rectangular volume element.